Cryptography 101: From Theory to Practice

Chapter 12 – Key Establishment

Rolf Oppliger

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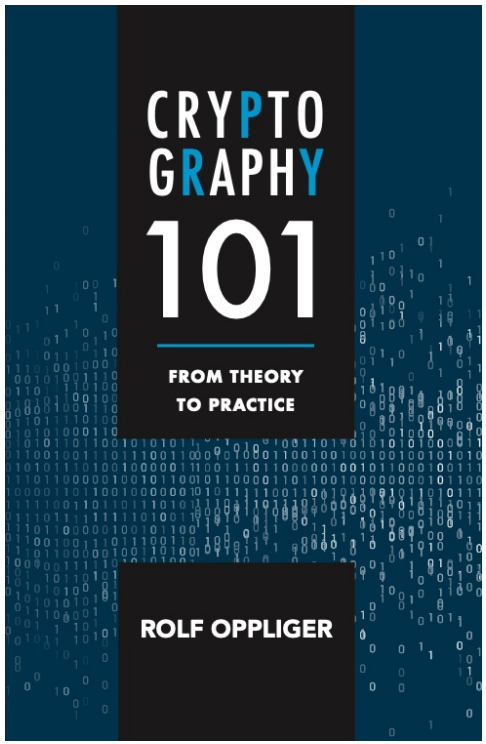
whoami

rolf-oppliger.ch
rolf-oppliger.com

- Swiss National Cyber Security Centre NCSC (scientific employee)
- eSECURITY Technologies Rolf Oppliger (founder and owner)
- University of Zurich (adjunct professor)
- Artech House (author and series editor for information security and privacy)

Reference Book



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Challenge Me

Part III

PUBLIC KEY CRYPTOSYSTEMS

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12. Key Establishment

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12. Key Establishment

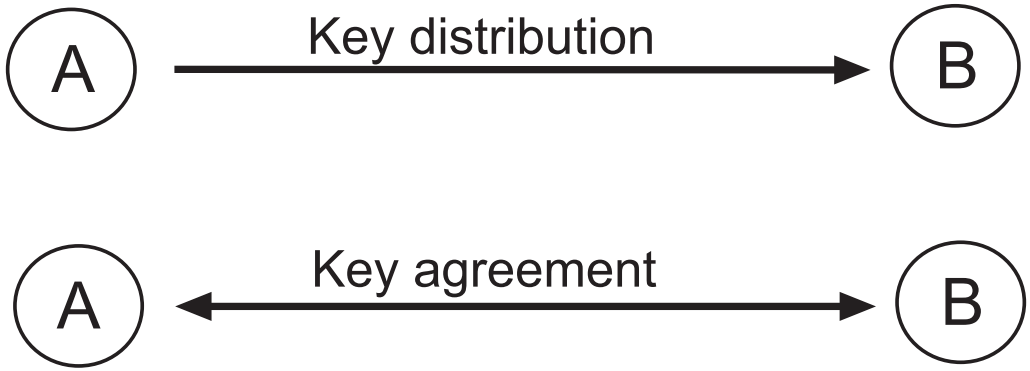
12.1 Introduction

- The establishment of keys is a major problem in secret key cryptography
- It represents the Achilles' heel for its large-scale deployment
- Two approaches
  - Key distribution center (KDC), such as Kerberos
  - Key establishment protocols

12. Key Establishment

12.1 Introduction

- Types of key establishment
  - Key distribution
  - Key agreement (aka key exchange)



12. Key Establishment

12.2 Key Distribution — Merkle's puzzles

- In 1975, Ralph C. Merkle proposed an idea that predates (but is conceptually similar to) public key cryptography
- It is theoretically interesting but not practical

Table 12.1
Merkle's puzzles

A		B
(n)		(n)
Generate $P_i = (i, k_i)$ for $i = 1, \ldots, n$ Permute $P_1, \ldots, P_n$		
	$\xrightarrow{P_{\pi(1)}, \ldots, P_{\pi(n)}}$	
		Randomly select P_i Solve P_i
	$\xleftarrow{i}$	
(k_i)		(k_i)

12. Key Establishment

12.2 Key Distribution — Shamir's three-pass protocol

- In 1980, Adi Shamir proposed a key distribution protocol that employs commutative encryption

Table 12.2

Shamir's Three-Pass Protocol (error in the printed edition of the book)

A		B
(n)		(n)
$k \xleftarrow{r} \mathcal{K}$		
	$\xrightarrow{k_1 = E_{k_A}(k)}$	
	$\xleftarrow{k_2 = E_{k_B}(E_{k_A}(k))}$	
$D_{k_A}(k_2)$		
	$\xrightarrow{k_3 = E_{k_B}(k)}$	
		$D_{k_B}(k_3)$
(k)		(k)

12. Key Establishment

12.2 Key Distribution — Shamir's three-pass protocol

- An additive stream cipher, such as the one-time pad, yields a commutative encryption
- In this case, however, all encryptions cancel themselves out and the protocol gets totally insecure
- If r_A is the bit sequence used by A to compute k_1 and k_3, and r_B is the bit sequence used by B to compute k_2, then k_1, k_2, and k_3 can then be expressed as

$$\begin{aligned} k_1 &= r_A \oplus k \\ k_2 &= r_B \oplus k_1 = r_B \oplus r_A \oplus k \\ k_3 &= r_A \oplus k_2 = r_A \oplus r_B \oplus r_A \oplus k = r_B \oplus k \end{aligned}$$

12. Key Establishment

12.2 Key Distribution — Shamir's three-pass protocol

- These are the values an adversary can observe in a passive (wiretapping) attack
- The adversary can add k_1 and k_2 modulo 2 to retrieve r_B

$$k_1 \oplus k_2 \quad = \quad r_A \oplus k \oplus r_B \oplus r_A \oplus k = r_B$$

- This value can then be added modulo 2 to k_3 to determine k

$$r_B \oplus k_3 = r_B \oplus r_B \oplus k = k$$

12. Key Establishment

12.2 Key Distribution — Shamir's three-pass protocol

- The bottom line is that a perfectly secure symmetric encryption system is used, and yet the resulting key distribution protocol is totally insecure
- This suggests that the use of an additive stream cipher is inappropriate to instantiate the three-pass protocol
- Shamir suggested the use of modular exponentiation in $\mathbb{Z}_p^*$ (instead of an additive stream cipher)
- The resulting three-pass protocol is known as **Shamir's three-pass protocol** (or **Shamir's no key protocol**)

12. Key Establishment

12.2 Key Distribution — Shamir's three-pass protocol

- Respective values for k_1, k_2, and k_3:

$$\begin{aligned} k_1 &\equiv k^{e_A} \pmod{p} \\ k_2 &\equiv (k^{e_A})^{e_B} \equiv k^{e_A e_B} \pmod{p} \\ k_3 &\equiv ((k^{e_A})^{e_B})^{d_A} \\ &\equiv ((k^{e_A})^{d_A})^{e_B} \\ &\equiv (k^{e_A d_A})^{e_B} \\ &\equiv k^{e_B} \pmod{p} \end{aligned}$$

- B can use d_B to retrieve k:

$$k \equiv (k^{e_B})^{d_B} \equiv k^{e_B d_B} \equiv k \pmod{p}$$

12. Key Establishment

12.2 Key Distribution — Shamir's three-pass protocol

- In 1982, James L. Massey and Jim Omura proposed the use of a binary extension field $\mathbb{F}_{2^m}$ for $m \in \mathbb{N}$ (instead of $\mathbb{Z}_p^*$)
- The resulting variant of Shamir's three-pass protocol is known as the **Massey-Omura protocol**
- It allows hardware implementations to be more efficient
- All known instantiations of Shamir's three-pass protocol employ modular exponentiation in one way or another (and therefore refer to public key cryptography)

12. Key Establishment

12.2 Key Distribution — Asymmetric encryption-based key distribution protocol

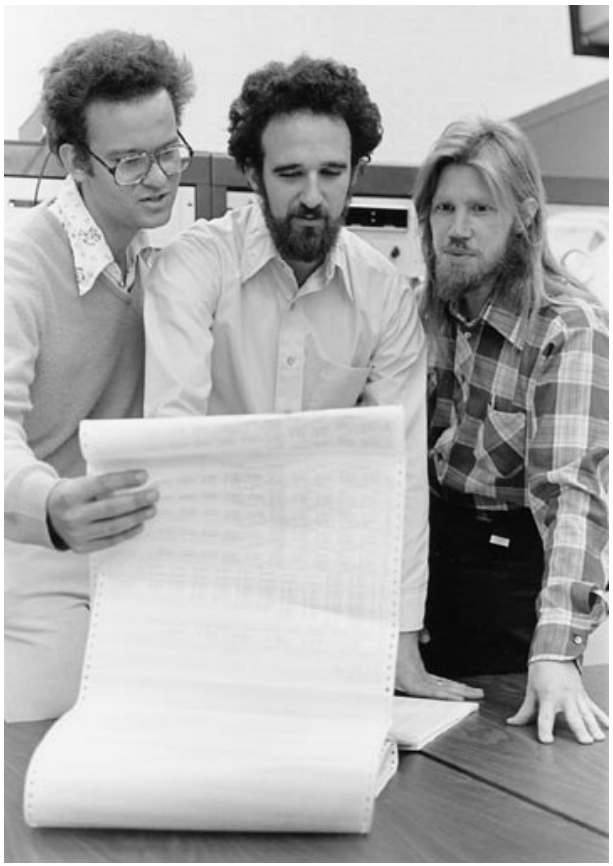
- Asymmetric encryption-based key distribution is used in many Internet security protocols (e.g., IPsec/IKE, SSL/TLS, . . .)

Table 12.3
An Asymmetric Encryption-based Key Distribution Protocol

A		B
(pk_B)		(sk_B)
$k \xleftarrow{r} \mathcal{K}$		
	$\xrightarrow{\text{Encrypt}(pk_B, k)}$	
		$k = \text{Decrypt}(sk_B, \text{Encrypt}(pk_B, k))$
(k)		(k)

12. Key Establishment

12.3 Key Agreement — Diffie-Hellman key exchange protocol



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- In 1976, Whitfield Diffie and Martin Hellman published a landmark paper entitled "New Directions in Cryptography"
- The paper introduced the notion of public key cryptography and proposed a key agreement protocol
- The paper changed the field
- Diffie and Hellman won the ACM A.M. Turing Award in 2015

12. Key Establishment

12.3 Key Agreement — Diffie-Hellman key exchange protocol

- The **Diffie-Hellman key exchange** (**exponential key exchange**) protocol can be used by two entities that have no prior relationship to agree on a secret key by communicating over a public but authentic channel
- As such, its existence seems paradoxical at first sight

Table 12.4
Diffie-Hellman Key Exchange

A		**B**
(G, g)		(G, g)
$x_a \xleftarrow{r} \mathbb{Z}_q^*$ $y_a = g^{x_a}$		$x_b \xleftarrow{r} \mathbb{Z}_q^*$ $y_b = g^{x_b}$
	$\xrightarrow{y_a}$	
	$\xleftarrow{y_b}$	
$k_{ab} = y_b^{x_a}$		$k_{ba} = y_a^{x_b}$
(k_{ab})		(k_{ba})

12. Key Establishment

12.3 Key Agreement — Diffie-Hellman key exchange protocol

- Toy example
  - $p = 23$ is a safe prime, because $11 = (23 - 1)/2$ is also prime
  - $\mathbb{Z}_{23}^* = \{1, \ldots, 22\}$ has a subgroup G that consists of the $q = 11$ elements 1, 2, 3, 4, 6, 8, 9, 12, 13, 16, and 18
  - $g = 3$ is a generator of this group (there are others)
  - A randomly selects $x_a = 6$, computes $y_a = 3^6 \mod 23 = 16$, and sends this value to B
  - B randomly selects $x_b = 9$, computes $y_b = 3^9 \mod 23 = 18$, and sends this value to A
  - A computes $y_b^{x_a} = 18^6 \mod 23 = 8$
  - B computes $y_a^{x_b} = 16^9 \mod 23 = 8$
  - The result is $k = k_{ab} = k_{ba} = 8$

12. Key Establishment

12.2 Key Distribution — Diffie-Hellman key exchange protocol

Table 12.5
A MITM Attack Against the Diffie-Hellman Key Exchange Protocol

A	**C**	**B**
(G, g)		(G, g)
$x_a \xleftarrow{r} \mathbb{Z}_q^*$		$x_b \xleftarrow{r} \mathbb{Z}_q^*$
$y_a = g^{x_a}$		$y_b = g^{x_b}$
	$y_a \rightsquigarrow y_c$	
	$y_c \rightsquigarrow y_b$	
$k_{ac} = y_c^{x_a}$		$k_{bc} = y_c^{x_b}$
(k_{ac})		(k_{bc})

12. Key Establishment

12.3 Key Agreement — Diffie-Hellman key exchange protocol

- The source of the problem (i.e., susceptibility to MITM attack) is the lack of authenticity
- People therefore prefer an **authenticated Diffie-Hellman key exchange**, such as provided by the station-to-station (STS) protocol
- There are many possibilities to authenticate a Diffie-Hellman key exchange using complementary cryptographic techniques, such as passwords, secret keys, or digital signatures and public key certificates
- In the case of a password, the **encrypted key exchange (EKE)** protocol yields an alternative

12. Key Establishment

12.2 Key Distribution — Diffie-Hellman key exchange protocol

Table 12.7
ECDH Protocol

A		B
$(Curve, G, n)$		$(Curve, G, n)$
$d_a \xleftarrow{r} \mathbb{Z}_n \setminus \{0\}$		$d_b \xleftarrow{r} \mathbb{Z}_n \setminus \{0\}$
$Q_a = d_a G$		$Q_b = d_b G$
	$\xrightarrow{Q_a}$	
	$\xleftarrow{Q_b}$	
$k_{ab} = d_a Q_b$		$k_{ba} = d_b Q_a$
(k_{ab})		(k_{ba})

12. Key Establishment

12.2 Key Distribution — Diffie-Hellman key exchange protocol

- Toy example
  - $(Curve, G, n)$ from Chapter 5, i.e., $a = b = 1$ $(y^2 \equiv x^3 + x + 1)$, $p = 23$ $(\mathbb{Z}_{23})$, $G = (3, 10)$, and $n = 28$
  - A selects $d_a = 6$, computes $Q_a = 6G = 6(3, 10) = (12, 4)$, and sends $Q_a = (12, 4)$ to B
  - B selects $d_b = 11$, computes $Q_b = 11G = 11(3, 10) = (18, 20)$, and sends $Q_b = (18, 20)$ to A
  - A computes $d_a Q_b = 6(18, 20) = (6, 4)$
  - B computes $d_b Q_a = 11(12, 4) = (6, 4)$
  - Note that $6 \cdot 11(3, 10) = 11 \cdot 6(3, 10) = 66 \bmod 28(3, 10) \equiv 10(3, 10) = (6, 4)$

12. Key Establishment

12.4 Quantum Cryptography

- **Quantum cryptography** refers to a key establishment technology that is based on the laws of quantum physics (instead of mathematics)
- More specifically, it makes use of the **Heisenberg uncertainty principle**
- This principle states that certain pairs of physical quantities of an object, such as its position and velocity, cannot both be measured exactly at the same time
- This has practical implications for the exceedingly small masses of atoms and subatomic particles, like photons

12. Key Establishment

12.4 Quantum Cryptography

- To measure the polarization of a photon, one can use one of two bases:
  - A **rectilinear basis** (⊞) is able to reliably distinguish between photons that are polarized horizontally with 0°/180° (↔) or vertically with 90°/270° (↕)
  - A **diagonal basis** (⊠) is able to reliably distinguish between photons that are polarized diagonally, i.e., with either 45°/225° (⤢) or 135°/315° (⤡)
- The two bases (i.e., ⊞ and ⊠) are **conjugate** in the sense that the measurement of the polarization in one basis randomizes the measurement in the other basis
- This can be exploited to establish a **quantum channel**

12. Key Establishment

12.4 Quantum Cryptography

- In 1984, Charles H. Bennett and Gilles Brassard proposed a quantum cryptography-based key exchange protocol known as **quantum key exchange (QKE)**
- A may send out photons in one of four polarizations: $\leftrightarrow$, $\nearrow$, $\updownarrow$, or $\nwarrow$
- B measures the polarizations of the photons it receives
- According to the laws of quantum physics, B can distinguish between rectilinear polarizations (i.e., $\leftrightarrow$ or $\updownarrow$ using $\boxplus$) and diagonal polarizations (i.e., $\nearrow$ or $\nwarrow$ using $\boxtimes$), but it cannot distinguish between both types of polarization simultaneously

12. Key Establishment

12.4 Quantum Cryptography

- Coding rules (encoding and decoding)
  - Rectilinear basis (⊞)
    - Horizontal polarization with 0°/180° (↔) stands for 0
    - Vertical polarization with 90°/270° (↕) stands for 1
  - Diagonal basis (⊠)
    - Diagonal polarization with 45°/225° (⤢) stands for 0
    - Diagonal polarization with 135°/315° (⤡) stands for 1

12. Key Establishment

12.4 Quantum Cryptography

Table 12.8
An Exemplary Transcript of the Quantum Key Exchange Protocol

1)	0	0	1	0	1	1	0	1	1	0
2)	⊞	⊠	⊠	⊞	⊠	⊞	⊞	⊠	⊠	⊞
3)	↔	⤢	⤡	↔	⤡	↕	↔	⤡	⤡	↔
4)	⊞	⊞	⊠	⊞	⊞	⊠	⊞	⊞	⊠	⊠
5)	0		1	0	*0*	*0*	0	*1*	1	*0*
6)	⊞		⊠	⊞	⊞	⊠	⊞	⊞	⊠	⊠
7)	✓		✓	✓			✓		✓	
8)	0		1	0			0		1	
9)			1				0			
10)			✓				✓			
11)	0			0					1	

12. Key Establishment

12.4 Quantum Cryptography

- Advantages
  - Provides an alternative key exchange method (whose security does not depend on mathematics)
  - Resistant to quantum computing
- Disadvantages (problem areas)
  - Requires specialized hardware (costs)
  - Requires and authentic channel (bootstrap problem)
  - Relatively small distances ($\approx$ 100 km)
  - Must be combined with "conventional" cryptography
  - ...

12. Key Establishment

12.4 Quantum Cryptography

- Many researchers have contributed to quantum cryptography in many ways
- In 1991, Artur Ekert proposed an alternative QKE protocol that uses entangled photons
- In addition to quantum key distribution protocols (using polarized or entangled photons), many other quantum cryptographic protocols have been developed and proposed
- A few companies sell quantum cryptographic devices and products, such as ID Quantique and MagiQ Technologies

12. Key Establishment

12.5 Final Remarks

- The Diffie-Hellman key exchange protocol is omnipresent (i.e., it is used in almost all Internet security protocols)
- Whenever two entities want to establish a secret key, it provides an elegant and highly efficient solution
- Using bilinear maps, it can be generalized to three entities
- The generalization to > 3 entities remains an open problem
- People use Diffie-Hellman trees to come up with key exchange protocols that have a complexity that grows "only" logarithmically with the number of entities (e.g., IETF MLS WG)

Questions and Answers

Thank you for your attention